170 and 180 miles—say 175. The most probable velocity therefore decidedly indicates a hyperbolic orbit.

Owing perhaps to the late hour at which the meteor appeared but few observations of the phenomenon were reported. Several letters of inquiry brought no available response, and Prof. Cleveland Abbe of the U. S. Signal Service informed me that no accounts of the meteor were received at the Washington Office.

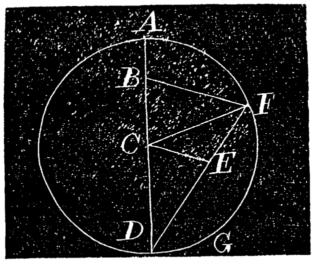
Bloomington, Indiana, Sept., 1878.

QUINQUISECTION OF THE CIRCUMFERENCE OF A CIRCLE.

BY PROF. L. G. BARBOUR, RICHMOND, KENTUCKY.

Theorem.—Let C be the centre of a circle; AD, a diameter. Divide AC in extreme and mean ratio, putting the larger segment next the centre. Then from D as a centre, with a radius equal to DB, describe an arc cutting the circumference in F.

The arc AF will be one fifth of the circumference.



Demonstration. - Join BF, CF and DF and draw CE parallel to BF

By hypothesis

$$AB : BC :: BC : AC.$$

By composition, because $AC = CD$,

$$CD : BC :: BD : CD;$$

$$\therefore \quad BC : CD :: CD : BD.$$

The triangles BCF and CFD, having the same altitude, are to each other as their bases; $\therefore$ $BFC : CFD :: BC : CD$. Similarly

$$CFD : BFD :: CD : BD.$$

But the last couplets of these proportions, themselves form a proportion;

$$\therefore \quad BFC : CFD :: CFD : BFD.$$

Also, since CE is parallel to BF, we have

$$DCE : CFD :: CFD : BFD.$$

Comparing this with the last proportion, we find $BFC = DCE$. These two triangles then are equal in area; the base DE of the one, is equal to the base CF of the other, for CED is isosceles, because similar to BFD, and hence $DE = CD$. Moreover, the angle DCE opposite DE is equal to the angle DBF. But any two triangles of equal areas, equal bases, and having the angles opposite the bases, equal, each to each, are equal in all

their parts. $\therefore$ angle BFC=angle at D. $BCF = CBF = DCE$. But BCF, being measured by the arc AF, is double the angle at D, which is measured by half the arc AF; $\therefore \angle BCF = 2 \angle BFC$.

The angles BFC, BCF and CBF are together equal to two right angles, $\therefore BCF = \frac{2}{5}$ of two right angles, or $\frac{1}{5}$ of four right angles; $\therefore AF$ is $\frac{1}{5}$ of the circumference. Q. E. D.

SCHOLIA.—1. Taking the angle D as the unit of measurement, we have $D = BFC = FCE = CFE$, $2D = FBC = FCB = BFD = DCE = DEC$; $3D = CEF = BCE = FCD = FBA$.

2. $DE = DC = CF = BF =$ radius; $BC = CE = EF =$ greater segment of radius. Also the triangles CDE, CEF, DCF, BCF and BFD, i. e., all the triangles in the figure, are isocceles.

3. Laying off from F towards D an arc equal to AF, we get a remainder DG which is $\frac{1}{10}$ of the circumference. We know from Euclid that the chord from D to G is equal to BC; hence BC, CE and EF are each equal to the chord of a decagon, and DE, &c., to the chord of a hexagon. Also, $BD = DF =$ chord of $\frac{3}{5}$ of two right angles, or $\frac{6}{5}$ of one right angle, $\therefore FCD = FBA$ &c. = angle between two consecutive sides of a regular pentagon.

4. Since $BC : CD :: CD : BD$, chord of a regular inscribed decagon : chord of hexagon :: chord of hexagon : chord of 108°.

Putting CD=radius $= 1$, chord $36° = \dfrac{1}{\text{chord } 108°}$; $\therefore \sin 18° = \dfrac{1}{\sin 54°}$.

Since $BD = CD + BC$, chord $108° = 1 +$ chord $36°$. $\therefore \sin 54° = .5 + \sin 18°$, as may be seen in a table of natural sines.

To find the length of BC, which we will call x, $x : 1 :: 1 : x+1$; $\therefore x^2 + x = 1$, whence $x = -\frac{1}{2} \pm \frac{1}{2}\sqrt{5} = BC$; $\therefore BD = DF = +\frac{1}{2} + \frac{1}{2}\sqrt{5}$.

$AF^2 = AD^2 - DF^2 = 4 - \frac{1}{4} - \frac{1}{2}\sqrt{5} - \frac{5}{4} = \frac{5}{2} - \frac{1}{2}\sqrt{5} = BC^2 + CD^2$; $\therefore$ the side of a regular inscribed pentagon is the hypothenuse of a right angled triangle, the other sides being the side of a regular inscribed hexagon and that of a regular inscribed decagon. This has been noticed by Young.

A PROBLEM AND ITS SOLUTION.

BY DR. H. EGGERS, MILWAUKEE, WISCONSIN.

Problem. — Given in a plane three fixed right lines, L_1, L_2, L_3, and in each of them a fixed point, respectively, A_1, A_2, A_3: required a right line M, which shall cut off on L_1, L_2, L_3, three equal distances counting from